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Application of a Genetic Algorithm to Wind Turbine Design

This paper presents an optimization method for stall-regulated horizontal-axis wind turbines. A hybrid approach is used that combines the advantages of a genetic algorithm with an inverse design method. This method is used to determine the optimum blade pitch and blade chord and twist distributions that maximize the annual energy production. To illustrate the method, a family of 25 wind turbines was designed to examine the sensitivity of annual energy production to changes in the rotor blade length and peak rotor power. Trends are revealed that should aid in the design of new rotors for existing turbines. In the second application, five wind turbines were designed to determine the benefits of specifically tailoring wind turbine blades for the average wind speed at a particular site. The results have important practical implications related to rotors designed for the Midwestern US versus those where the average wind speed may be greater.

Introduction

Genetic algorithms (GAs) are robust parameter search techniques based on the Darwinian concept of natural selection (Holland, 1992; Goldberg, 1989). Unlike calculus-based optimization algorithms that iterate on a single solution toward the optimum, GAs iterate on a set of solutions. Each solution is termed an individual, and each set of solutions is referred to as a generation or population. Iteration is performed from one generation to the next. Every individual in a generation is assigned a fitness that corresponds to its performance based on the objective function defined for the particular problem. An individual that performs well for a given generation is assigned a high fitness. Characteristics from the fittest individuals are then used to help define the next generation of individuals. This approach to determining subsequent generations is the computational analog to the concept of "the survival of the fittest." This proven technique provides an efficient parameter search algorithm that exploits information from previous generations to create new individuals with expected improved performance.

Application of genetic algorithm techniques to the conceptual design of horizontal-axis wind turbines (HAWTs) is especially attractive. In the past, wind turbine design has been performed largely by the direct approach or design-by-analysis. If several design variables are considered during the design process, the task of performing a sufficient number of trade studies can be daunting. This parameter search for the optimum rotor involves complex trade-offs between, for instance, the blade chord and lift distribution, the rotor radius and rpm, the blade pitch and peak rotor power characteristics, to name just a few. This laborious design process can be efficiently automated with a genetic algorithm.

Unfortunately, simple GAs were developed to solve unconstrained optimization problems. Constrained problems, both equality and inequality, can be transformed into unconstrained problems with the use of penalty functions (Goldberg, 1989). However, for highly constrained problems, the efficiency of the GA is reduced. In the current method, a novel approach to constraint enforcement is taken. An HAWT inverse design scheme (Selig and Tangler, 1995) is used to satisfy nonlinear constraints. The use of an inverse method to achieve desired constraints is particularly attractive since the designer has the flexibility to specify either independent or dependent variables (e.g., rotor geometry or aerodynamic performance characteristics). For instance, in the examples given later, the inverse method is used to enforce a peak rotor power constraint, which is present when a stall-regulated wind turbine is designed for a fixed generator. Generally, the embedded inverse scheme can be used to satisfy specified rotor aerodynamic characteristics that would otherwise be achieved through a penalty function.

The GA offered two major advantages over the more traditional gradient-based optimization methods. First, as the name implies, gradient-based algorithms require the user to provide derivatives of the objective function (annual energy production) with respect to the design variables (blade chord, twist, and pitch.) These expressions are difficult to calculate analytically and can be computationally burdensome to estimate numerically. No gradient information is required for the GA optimization technique. The second reason for using the GA is its ability to effectively search a parameter space that may have many local optimal solutions (blade designs). Gradient-based algorithms converge to the nearest local optimal design, while GAs have the ability to compare these locally optimized designs and thereby select the superior one.

The remainder of this paper describes the current design methodology that integrates the advantages of a genetic algorithm with an inverse design technique for HAWTs. The method is subsequently applied to the design of a family of optimum stall-regulated wind turbines to assess the effects of changing the rotor blade length and peak rotor power. The blade length and peak power range used in the study is applicable to a wide range of wind turbines in current use.

The Design Methodology

Genetic Algorithm. As background, genetic algorithms were first developed by John Holland and his students at the University of Michigan in the 1970s. Since their conception, GAs have gained in popularity and have been used to solve optimization problems in many diverse subjects, such as engineering, science, economics, and finance. Very recently, Holland (1992) proved theoretically and empirically that GAs provide a robust search method in complex spaces. For this study, a GA is developed to design optimal wind turbines. The GA

Contributed by the Petroleum Division for publication in the JOURNAL OF EN-ERGY RESOURCES TECHNOLOGY. Manuscript received by the Petroleum Division, November 28, 1994; revised manuscript received October 31, 1995. Associate Technical Editor: H. Richter.

differs from typical search methods in the way that the design variables are represented and the way that the algorithm searches the parameter space.

Design Variable Representation. The GA converts a continuous parameter space that defines the wind turbine into a finite length string of characters. The individual parameters are coded and linked into a chain that contains the characteristics of the wind turbine in much the same way that a DNA chain contains biological characteristics. The GA operates by manipulating the finite length string, not the parameters themselves. In this paper, the parameters (chord, blade pitch, and twist) are represented by a binary string.

For an example of binary coding, suppose that the search range for a parameter is set between -1 and +1 and that a 10bit string is used to represent the parameter. A 10-bit string is capable of representing 210 (1024) numbers. With this code, the string 000000000 would represent the number -1 and 1111111111 the number 1 with a linear mapping between -1and 1. For this study, each parameter is characterized by a 10bit string. The parameter strings are linked to form a single individual. If two parameters with 10-bit length string are combined, the single string represents one of 2^{20} (1,048,576) possible combinations. For the HAWT designs, a total of eight design parameters, each represented by a 10-bit string, were requiredan 80-bit binary string. Fortunately, all 2^{80} (1.2 × 10²⁴) possible solutions do not need to be checked to determine the optimal solution, but only a subset of these combinations are involved in the search technique.

Search Technique. The GA differs from traditional optimization techniques in several ways. First, the parameter space is searched with a group or population of points, not a single point. Each point represents a possible solution to the problem (a particular wind turbine design). The direction that each individual or point in the population moves depends on its fitness, or measure of goodness, relative to other individual fitness. In this paper, the fitness that is to be maximized is the annual energy production of the particular wind turbine design.

To initiate the GA search, the first generation is created by randomly forming character strings of a given length. Each string or individual represents a possible solution to the optimization problem and is assigned a number corresponding to its annual energy production. If an individual performs well, then it is given a high fitness rating. The higher an individual's fitness, the larger the probability that this individual will reproduce (survival of the fittest) and its characteristics, some of which must be good, propagate to the next generation.

The manner in which the GA iterates from one generation to the next involves the use of probability and entails the operation of reproduction, crossover and mutation. The probabilistic transition rules differ from traditional methods in that the conventional approaches are typically deterministic being based on gradients of the objective function. With a GA, there are several methods to model the transition from one generation to the next, referred to as reproduction. The wind turbine design GA uses simple tournament selection. This reproduction method randomly selects two individuals from the current generation and compares their associated fitness. The individual with the higher fitness is allowed to reproduce. The tournament selection continues until the appropriate number of parents have been identified. Next, a crossover of information occurs. Two parents are randomly chosen to be paired. If crossover of information is to happen at a given location, based on the probability of crossover, that particular bit in each parent is swapped. For example, consider the following 3-bit binary string parents:

111

000

Suppose that bits i and 3 are chosen as crossover sites. The two children that result from the crossover are

010

101

Mutation is the third probabilistic operator. Mutation involves the switching of a randomly selected bit from a 1 to a 0 or vice versa. The occurrence of mutation, based on the probability of mutation, models nature's ability to have species evolve beneficial characteristics over several generations. Mutation also provides a mechanism to avoid getting trapped in a local extremum. Finally, elitism is also used in this design approach. The elitist model ensures the survival of the most fit individual from one generation to the next without altering its string representation.

Inverse Design Algorithm. The inverse scheme embedded into the GA is that discussed in Selig and Tangler (1995). The method as a stand-alone design tool can be used to directly specify desired aerodynamic characteristics from which the corresponding blade geometry is determined. More generally, any desired physically realizable characteristics can be achieved so long as some variables are left to be determined (e.g., blade chord and twist distributions, pitch, rpm-traditional input variables.) The degree to which the inverse capability is used remains the designer's decision. Once incorporated into the GA, the inverse scheme can be used to achieve desired rotor constraints. In order to achieve these constraints, some design parameters must be adjusted through iteration. Thus, some design parameters are assigned for iteration in the inverse scheme, while others are assigned for iteration in the GA. The method is highly flexible in that the designer may select certain variables for iteration. Thus, the approach retains some of the advantages traditionally associated with the design-by-analysis approach; that is, some design parameters may be controlled by the designer. As mentioned in the Introduction, the inverse method is used to satisfy the rotor peak power constraint.

Hybrid Algorithm. The genetic algorithm with the embedded inverse scheme to enforce constraints is integrated into the computer program: PROPGA. A simplified flow diagram for PROPGA is depicted in Fig. 1. The designer specifies the parameters that the GA is to determine and also the parameters that the embedded inverse design code is to determine. The method searches from a population of individuals, as mentioned before. The first generation is populated with randomly created individuals, while subsequent generations are created from individuals that performed well in the preceding generation. Each individual is composed of the GA selected design parameters that are then input into the inverse design algorithm. Any constraints on the design are satisfied by varying the inverse design selected parameters. Once all the design variables have been determined, the resulting wind turbine performance is evaluated and assigned a fitness. The fitness (e.g., annual energy production) represents a measure of goodness that will be used in creating subsequent generations. The process continues until each wind turbine in the generation is designed and analyzed for its corresponding fitness. Generations of wind turbines are created until the desired number of generations have been developed, at which time the algorithm is terminated.

Application

The wind turbine optimization method described in the foregoing has been applied to the design of several stall-regulated wind turbines. First, a family of 25 wind turbines was designed to examine the sensitivity of annual energy production to changes in the rotor blade length and peak rotor power. For this group, the rotors were optimized for an average wind speed of 7.15 m/s (16 mph). Second, a series of five wind turbines was designed to study the change in annual energy production to changes in the average wind speed for the wind farm. This particular question is addressed since it is of considerable inter-



Fig. 1 Flow diagram for PROPGA

est to know if rotors should be uniquely designed, depending on the average wind speed at a wind farm site. For this second study, the peak rotor power and blade radius were fixed, while the wind farm average wind speed was changed from 5.36m/s (12 mph) to 8.94 m/s (20 mph). The results are then compared with the baseline rotor that was optimized for an average wind speed of 7.15 m/s (16 mph).

Since the main goal was to explore the effects of changing the rotor radius, peak power, and wind farm average wind speed, several fixed constraints were imposed. All rotors were threebladed and operated at a rotor speed of 44 rpm. The NREL airfoils—S814 (root), S809 (primary, 75 percent radius location) and S810 (tip)—were incorporated into the current blade design and offer the following advantages (Tangler, 1987; Tangler et al., 1992):

- restrained c_{t,max} tip airfoils for peak power control with optimum blade radius, increased annual energy production, and lower wind farm array losses;
- reduced roughness sensitivity for improved energy capture under dirty blade conditions;

 increased section thickness (S814-24, S809-21, and S810-18 percent) for lower blade weight, lower cost, increased stiffness, and improved fatigue resistance.

As mentioned, for all rotors, the rpm was fixed in order to reduce the number of design variables to a managable number. In a more exhaustive trade study, however, an examination of the effect of rpm would be prudent since gearboxes can be supplied with a variety of gear ratios.

For each wind turbine presented, PROPGA was used to define the optimum blade pitch and chord and twist distributions for maximum annual energy production at a specified average wind speed. Specifically, the genetic algorithm was used to define the optimum blade pitch and relative chord and twist distributions shown in Fig. 2, which requires some discussion. The chord c is composed of the sum of a constant level at the blade root and a chord distribution relative to this constant level, that is, $c_0 + \bar{c}$. Similarly, the blade pitch is the sum of the pitch β at 75 percent radius and the twist θ relative to this point, both measured positive in the direction toward feather from the rotor plane. The optimum relative chord distribution and the twist distribution were defined by movable spline supports along the blade. A total of seven parameters (four chord and three twist locations) were used and converted into a binary representation and then concatenated into a single coded string (i.e., an individual). The inverse method was used to satisfy the prescribed peak rotor power constraint, which was achieved by iterating on the rotor-blade chord offset c_0 .

To determine the optimum blade geometry, 45 generations were designed. Each generation was made up of 100 individuals. The probabilities associated with crossover and mutation were 60 and 1.25 percent, respectively. For each generation, not all of the 100 individuals were successfully designed. Throughout the optimization process it was possible for the chord at a particular point to be negative as defined by the GA or become negative during iteration on the chord offset c_0 in the inverse scheme. For such an individual, the fitness was set to zero, and consequently, the characteristics of that individual had a very small probability of progatating on through subsequent generations. Typically, only 10–40 percent of the individuals were successfully designed during the first generation. After 45 generations, however, 90–100 percent of the individuals were successfully designed.

The analysis method currently used in PROPGA is that extracted from the PROP code, which is based on blade-element/ momentum theory. It should be noted, however, that the PROP



Fig. 2 Parameterization of blade twist θ and chord c distributions as a function of blade radius r

Table 1 Rotor radius/peak rotor power trade study case matrix

•		13.00	13.50	14.00	14.50	15.00				
		Radius (m)								
		42.65	44.29	45.93	47.57	49.21				
		Radius (ft)								
	275.0	1-5	2-5	3-5	4-5	5-5				
Peak Power	262.5	1-4	2-4	3-4	4-4	5-4				
	250.0	1-3	2-3	3-3	4-3	5-3				
(kW)	237.5	1-2	2-2	3-2	4-2	5-2				
	225.0	1-1	2-1	3-1	4-1	5-1				

model currently employed could readily be replaced by a more computationally intensive, higher-fidelity rotor vortex code if the design cycle time is not critical and if there exists sufficient computer resources. Briefly, the earliest version of PROP was developed nearly two decades ago (Wilson and Lissaman, 1974; Wilson and Walker, 1974; Wilson et al., 1976; Perkins, 1979), and after various improvements (Hibbs and Rudkey, 1983; Tangler, 1983, 1987), it has become an industry standard. Desirable features of the code are that it allows for rapid analysis, accommodates different airfoil data for each blade element, and includes a three-dimensional post-stall airfoil performance synthesization method for better peak power prediction at high wind speed (Ostowari and Naik, 1984, 1985; Viterna and Corrigan, 1981). Although discrepancies between field tests and predictions have been documented and discussed, the code has proved to be invaluable as a guide to the design of a series of successful wind turbine blade designs. The annual energy production is predicted by the NREL SEACC code (Tu and Kertesz, 1983). The SEACC predictions were based on a Weibull wind speed distribution with shape factor K of 2, which corresponds to a Rayleigh distribution.

Optimum Wind Turbines for Varying Rotor Radius and Peak Power. In this trade study, the rotor radius and peak rotor power were varied from 13-15 m and 225-275 kW, respectively. This range was selected since several wind turbines that are in current operation fit within this scope, and many of these wind turbines are likely candidates for blade replacement. The results should therefore prove both useful and timely.

For the size range considered, 25 different blade radius/peak power combinations were examined as listed in Table 1. A special notation is used to denote each case. For example, case 2-4 refers to a rotor with a radius of 13.5 m (44.29 ft) and a peak power of 262.5 kW. In general, the first digit for a case description refers to the radius, and the second digit refers to the peak power—the larger the number, the larger the radius/ power.

As previously outlined, PROPGA was used to determine the optimum wind turbine for each case in the matrix (Table 1). Again, the objective was to maximize the annual energy production for an average wind speed of 7.15 m/s (16 mph). Figure 3 shows the optimum blade chord distributions for the corner cases (1-1, 1-5, 5-1 and 5-5). The trends are easily observed. As the peak power is increased, the blade area increases for a given rotor radius. For constant peak power, however, the blade area is reduced with increased radius. As the blade is extended, more power is produced by the blade tip region. To compensate for the increased power, the blade area must be reduced which explains the trend of decreasing area with increasing blade radius. It is interesting to note that the rotor planforms are much like those predicted from the classical optimal rotor theories (Glauret, 1935; Wilson and Lissaman, 1974). The twist distributions are consistent with these classical theories.

The corresponding optimum blade twist distributions are shown in Fig. 4. As seen, an interesting trend is observed—the



Fig. 3 Optimal blade chord distributions for (a) cases 1-1 and 1-5, and (b) cases 5-1 and 5-5

longer the blade radius, the greater the inboard twist toward stall. This result would be more apparent if the twist distributions were plotted versus the nondimensional blade radius. The inboard twist in the direction toward stall as the radius is increased indicates that the inboard blade generally operates at higher local lift coefficient. Consequently, stall occurs sooner for the larger radius, which (in addition to the reduced area) helps to regulate the peak rotor power.

The optimum blade pitch is shown as a contour plot in Fig. 5. For the most part, two trends are apparent. For a fixed peak power, the blade is pitched more toward stall as the blade radius is increased. This pitch toward stall (together with the reduced blade area and increased twist toward stall) is necessary to regulate the peak rotor power. Conversely, for a fixed blade radius, the blade is pitched more toward feather as the peak rotor power is increased. Lines of constant pitch show sharp corners partly because higher grid resolution resulted in spurious and misleading contours. Moreover, the airfoil data lift coefficient, drag coefficient and angle of attack were not smooth enough to provide high-fidelity contours. Nevertheless, the trends are clearly exhibited.

Figure 6 shows the maximum power coefficient $(C_{p,\text{max}})$ for each rotor. It is not surprising (in view of the definition for the power coefficient) to see the lowest value associated with the



Fig. 4 Optimal blade twist distributions for (a) cases 1-1 and 1-5, and (b) cases 5-1 and 5-5



Fig. 5 Optimum blade pitch setting

lowest peak power/largest rotor case and the highest value for the highest peak power/smallest rotor case. The rather high $C_{p,max}$ can be attributed to performance predictions based on the use of the blade-element/momentum theory approach, which does not fully capture the tip-loss effects. Consequently, the rotor power is overpredicted, as is the C_p .

Power curves for the corner cases are shown in Fig. 7. Moreover, contour plots of the annual energy are shown in Fig. 8 for the average wind speeds of 5.36 m/s (12 mph), 7.15 m/s (16 mph), and 8.94 m/s (20 mph). From the power curves (Fig. 7), it can be seen that as the peak power increases, so does the power curve and with it the annual energy. This trend is best seen in the contour plots (see Fig. 8). Also, as the rotor radius increases, the power and annual energy increase. Thus, as seen in the annual energy contour plots, lines of constant annual energy are typically inclined downward from left to right.

An important result to emerge is that to maximize annual energy production for a low-speed wind site it is beneficial to increase the rotor radius more than the peak rotor power. This result can be seen by realizing that at any point in the blade radius/rotor peak power design space (i.e., for any case), the path to take for the greatest increase in annual energy is normal to the lines of constant annual energy. This path inclines more toward increasing the blade radius as the wind speed is decreased.

The conclusion that for a low-speed wind site it is beneficial to increase the rotor radius more than the peak rotor power is in agreement with that presented in Selig and Tangler (1995). In that study, an optimization method was not used. Rather, lift







Fig. 7 Rotor power curves for (a) cases 1-1 and 1-5, and (b) cases 5-1 and 5-5

distributions and axial inflow distributions thought to be favorable were specified and the corresponding geometry was determined through the used of an inverse design method (PROPID).

It should be noted that in the current trade study (and in that of the following), the blade pitch was not specified, but rather it was determined as part of the optimum solution. In most cases, the resulting blade pitch is inclined more toward stall than desirable from a practical standpoint. Blade loads on stallregulated turbines can be high, and one way of mitigating these loads is to design the blade from the outset by fixing the blade pitch toward feather to a degree more than is presented here. In particular, a favorable blade pitch for the NREL series airfoils is believed to be 2-3 deg (Tangler, 1994). Consequently, the family of rotors (and the series to follow) should be considered as a special class of rotors optimized for maximum annual energy production subject to those constraints previously specified.

Optimum Wind Turbines for Varying Wind Farm Average Wind Speed. In this second study, a series of five wind turbines was designed, each for a different average wind speed, in particular, for average wind speeds from 5.36 m/s (12 mph) to 8.94 m/s (20 mph) in increments of 0.89 m/s (2 mph). The objective was to determine how advantageous it is to specially tailor the wind turbine for a particular wind farm site under the constraints of constant radius and peak power. Specifically, for all five rotors, the rotor radius and peak power were fixed at 14 m and 250 kW, respectively. Thus, the wind turbine optimized for an average wind speed of 7.15 m/s (16 mph) corresponds to case 3-3 in the previous trade study.

Figures 9 and 10 show the chord and twist distributions for the baseline case (previous case 3-3) and the wind turbines optimized for an average wind speeds of 5.36 m/s (12 mph) and 8.94 m/s (20 mph). The two other intermediate cases did not substantially deviate from the trends shown. The optimum blade pitch setting for the wind speeds of 5.36 m/s (12 mph), 7.15 m/s (16 mph), and 8.94 m/s (20 mph) were determined to be 0.97, 0.59, and 0.72 deg, respectively.



Fig. 8 Annual energy production for (a) 5.36 m/s (12 mph), (b) 7.15 m/s (16 mph), and (c) 8.94 m/s (20 mph)

Radius (m)

Power curves for the baseline and the two bounding cases are shown in Fig. 11. Only slight differences exist. The curves correspond to the wind turbines optimized for the average wind

Blade Chord Distributions for 14 m Radius Rotors



Fig. 9 Blade chord distributions for baseline rotor (7.15 m/s, case 3-3), and optimum rotors for different average wind speeds

Blade Twist Distributions for 14 m Radius Rotors







Fig. 11 Power for baseline rotor (7.15 m/s, case 3-3), and optimum rotors for different average wind speeds

speeds of 5.36 m/s (12 mph), 7.15 m/s (16 mph), and 8.94 m/s (20 mph), but are plotted as a function of the actual wind speed. Table 2 shows a comparison of the annual energy production between the five optimum wind turbines each analyzed at their respective design wind speed and the baseline rotor at the design point of 7.15 m/s (16 mph) and off-design at the four other wind speeds. As seen in Table 2, although the optimum case is an improvement over the baseline (except when the baseline is the optimum), the differences in annual energy production between the baseline rotor and optimum rotors are negligible.

The result that a specially designed wind turbine rotor for a low wind-speed site offers little (negligible) advantage over a wind turbine rotor designed for a higher wind-speed site clearly has important implications. Under the constraints of fixed radius and peak power, wind turbines designed for high wind-speed sites, such as those in California, are likely to be equally well designed for other sites, such as those in the Midwestern US where wind speeds are lower.

Conclusions

A new approach to the designed of horizontal-axis stall-regulated wind turbines and its application has been presented. The novel technique demonstrates that the robust parameter search

Table 2 Baseline rotor and optimum rotor annual energy comparison

		5.36	6.26	7.15	8.05	8.94		
		Average Wind Speed (m/s)						
		12.00 14.00 16.00 18.00 20.00						
		Average Wind Speed (mph)						
Annual	Baseline	553	727	84911	918	942		
Energy†	Optimum	556	729	849	919	943		
+ MM Llas								

† MW-Hrs

tt Design condition for the baseline

properties of a genetic algorithm together with an embedded inverse design technique can provide the ability to efficiently enforce desired rotor performance constraints during the optimization process. Application of the hybrid method (PROPGA) has yielded valuable and timely information related to the wind turbine blade design.

A trade study was performed to examine the sensitivity of annual energy production to changes in the rotor radius (13-15 m) and peak power (225-275 kW). To maximize annual energy production for a low-speed wind site, the results show that it is more beneficial to increase the rotor radius rather than the peak power-a conclusion consistent with an earlier finding. A second trade study was performed in which the rotor radius and peak power were held constant while optimizing the blade for different average wind speeds (5.36-8.94 m/s). The series of optimum blades were then compared with a baseline rotor designed for a single average wind speed (7.15 m/s). Although in each case the optimum wind turbine was an improvement over the baseline, the differences in annual energy production between the baseline rotor and optimum rotors were negligible. Consequently, if the blade radius and peak power are fixed constraints, wind turbine blades designed for highspeed wind sites, such as those in California, are likely to be equally well designed for lower speed sites, such as those in the Midwestern US.

Acknowledgments

The support of the National Renewable Energy Laboratory under Subcontract XAF-4-14076-03 is gratefully acknowledged. Also, the several discussions with James L. Tangler of NREL proved to be quite helpful during the course of this work.

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